INDIAN STATISTICAL INSTITUTE, BANGALORE CENTRE
B.MATH - Second Year, Second Semester, 2020-21
Statistics - II, Backpaper Examination

Answer all questions **Maximum Marks: 50**

Time: 3 hours; submission must be complete by 1:30 pm

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You may freely consult the lecture notes, but no other books or resources may be consulted. You may use any of the results stated and discussed in the lecture notes, by stating them explicitly. Results from the assignments may not be used without establishing them.

1. Let $X_1, \ldots, X_n$ be independent random variables with densities

$$f_{X_i}(x_i|\theta) = \begin{cases} \frac{1}{2i\theta} & \text{if } -i(\theta-1) < x_i < i(\theta+1); \\ 0 & \text{otherwise,} \end{cases}$$

$1 \le i \le n$, where $\theta > 0$.
(a) Find a two-dimensional sufficient statistic for θ.
(b) Find the maximum likelihood estimator of θ. **[7+8]**

2. Consider a random sample from $N(0, \sigma^2)$.
(a) Find the UMVUE of σ.
(b) Show that the UMVUE of σ is a consistent estimator.
(c) Find the asymptotic distribution of the UMVUE of σ. **[5+3+4]**

3. Suppose $X_1, X_2, \ldots, X_n$ is a random sample from Poisson(λ). Consider testing

$$H_0 : \lambda \le 1 \text{ versus } H_1 : \lambda > 1.$$

(a) Show that the conditions required for the existence of a UMP test are satisfied here.
(b) Derive the UMP test of level α. **[3+5]**

4. A large shipment of parts is received, out of which 5 are tested for defects. Let X denote the number of defective parts in the sample, and θ be the proportion of defective parts in the population. From past shipments it

is known that θ has a Beta(1, 9) distribution.
(a) Find the HPD estimate of θ if $x = 0$ is observed.
(b) Find a 95% credible set for θ if $x = 0$ is observed.
(c) For testing $H_0 : \theta \leq 0.10$ versus $H_1 : \theta > 0.10$, find the posterior odds ratio. **[5+5+5]**